International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Ingénierie des Systèmes d'Information
Vol. 28, No. 6, December, 2023, pp. 1695-1700
Journal homepage: http://iieta.org/journals/isi

Signal Processing Algorithms for Mean Square Error Analysis in MIMO Wireless Transceivers

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https://doi.org/10.18280/isi.280628

Received: 12 October 2023
Revised: 1 December 2023
Accepted: 7 December 2023
Available online: 23 December 2023

Keywords:
mean square error, least squares, minimum mean square error, maximum likelihood, multiple input multiple output, transceiver

ABSTRACT

Signal processing algorithms are crucial for the integrity of information transfer in wireless transceivers, with mean square error (MSE) serving as a pivotal metric for performance assessment. In multiple input multiple output (MIMO) systems, the transmission chain is susceptible to errors regardless of the antenna count, necessitating robust error analysis. This research article presents an evaluation of MSE in the context of MIMO wireless transceivers, focusing on data transmission at physical layer level. Signal processing algorithms, including least squares (LS), minimum mean square error (MMSE), and maximum likelihood (ML) algorithms, are analyzed for their efficacy in mean square error quantification, offering valuable insights for future research. A comprehensive analysis is conducted using training signals to ascertain the MSE, with simulations performed in MATLAB environment. Comparative results demonstrate that MMSE and ML algorithms outperform LS in reducing MSE, attributable to their reliance on probabilistic density functions (PDFs). The findings underscore the potential in error assessment and can aid emerging 5G and 6G wireless systems, which are predicated on advanced technologies such as massive MIMO and millimeter-wave communications. These results may pave the way for further research into optimizing signal fidelity in next-generation wireless communication systems.

1. INTRODUCTION

Signal processing is an integral component in the chain of information transfer for wireless transceivers. This field classifies signals by dimensionality: one-dimensional (1D) signals such as speech, two-dimensional (2D) signals like images, and three-dimensional (3D) signals, which include video. These signals, when contaminated by additive white Gaussian noise (AWGN) within a wireless transceiver, necessitate sophisticated processing algorithms for accurate information retrieval. Irrespective of the transceiver's system model or design specifications, the use of pilot signals [1] is essential for the efficient transfer of information. Pilot signals, which act as known reference signals for transmitters and receivers, enable the accurate processing of transmitted signals and the determination of mean square error (MSE) [2]. This determination is critical for managing data transmission and reception, whether through a single antenna system or a multiple input multiple output (MIMO) wireless transceiver [3]. MIMO systems, in particular, are known for their potential to provide increased diversity gain, spectral efficiency, reliability, and coverage in multipath propagation contexts [4].

Signal processing in MIMO systems [5] is considered at various levels, from architectural design in the radio frequency (RF) and in baseband stages. At the baseband level, digital modulation, such as orthogonal frequency division multiplexing (OFDM), is prominent in current 5G technology and anticipated 6G developments utilizing massive MIMO methodologies [6, 7]. The employment of pilot signals, constructed from discrete Fourier transform (DFT) matrices [8], facilitates the estimation of broadcast channel conditions through MSE when transmitted across wireless channels.

MSE is further attributed to it statistical components mean and variance. Here, the mean is the first moment of a random signal, and variance represents the power level of a signal corrupted by noise. Signal processing algorithms, including least squares (LS), minimum mean square error (MMSE), and maximum likelihood (ML) approaches [9], are utilized to estimate pilot signals within MIMO wireless transceivers. The LS algorithm minimizes error at the signal level by employing an optimization function. In contrast, MMSE and ML approaches rely on conditional probability density functions and likelihood functions for parameter estimation.

At the transmitter end, signal processing begins with the conversion of information from analog to digital form. This digital data undergoes source coding through Huffman and Shannon-Fano coding to minimize redundancy, followed by channel coding for error control [10]. Precoding schemes are also integrated at the MIMO transmitter to facilitate data transfer over wireless channels. The receiver end employs decoding and Bayesian detection to recover digital data signals [11], and ML based algorithms are used to minimize error in stationary Gaussian sources [12].

The research article [13] explores applications of MSE in advanced communication scenarios, including MIMO millimeter-wave communications, reconfigurable intelligent surface (RIS) enhanced multiuser MIMO systems in [14], and beamforming optimization [15]. In the network layer aspect of 5G systems, MSE analysis is crucial for maintaining quality of service (QoS) in heterogeneous networks and optimizing signal processing for multiuser MIMO uplink channels [16, 17]. It also contributes to spectral efficiency in multicell networks with imperfect channel state information (CSI) [18] and is essential for the joint design of MIMO and visible light communication (VLC) systems [19].

The necessity for robust wireless channel values is evident as it significantly impacts the efficacy of signal processing algorithms, which are assessed by the MSE metric. This research provides a comprehensive MSE analysis, along with different MIMO antenna configurations in Rayleigh and Rician fading channels.

The structure of this research article is as follows: Section 1 introduces the fundamentals of signal processing for MIMO wireless transceivers. Section 2 mathematically formulates the system model for MIMO wireless transceivers. Section 3 discusses the signal processing algorithms employed in MIMO wireless transceivers, including LS, MMSE, and ML approaches. Section 4 presents the simulation results for the MSE metric in MIMO wireless transceivers. Section 5 concludes the paper.

In this research article, matrices are denoted by boldface capital letters, vectors are indicated by boldface lowercase letters, and scalars are presented in standard font. Hermitian, transpose, complex conjugate, expectation, pseudo-inverse, inverse, trace, and the Frobenius norm of a matrix 'X' are represented as: x^H, x^T, x^*, $E\{.\}$, x^+, x^{-1}, trace{}, $\| \quad \|_F^2$ in this research paper.

2. MIMO WIRELESS TRANSCEIVER SYSTEM MODEL

The mathematical model of MIMO wireless transceiver system as per Figure 1 comprises of T_t transmitter antennas considered as an arrangement from transmitter (Tx) terminals such as *A, B, C, D, E* and a receiver base station (RXBS) comprising of T_r receiver antennas forming the representation of MIMO testbed. Each of the Tx terminals are equipped with omnidirectional antennas and transmit the training signals through the multipath fading channel to the RXBS. The training signal $\boldsymbol{d}$ is T_t×1 all unity vector sequence from discrete Fourier transform (DFT) matrix is broadcasted. The MIMO channel matrix $\boldsymbol{C}$ has dimensions T_r×T_t and observed at the RXBS is considered to be independent identically distributed (iid) with Gaussian distribution with mean μ_x and variance σ_x^2. The mathematical channel model can be modelled as Rayleigh channel or Rician channel at baseband level. The received signal vector at the *RXBS* i^{th} antenna at any given time slot is given as r_i. Further, the MIMO received signal matrix at the RXBS is given as:

$$\boldsymbol{R} = \boldsymbol{CD} + \boldsymbol{N} \tag{1}$$

where, $\boldsymbol{C}$ is T_r×T_t MIMO channel matrix and $\boldsymbol{D}$ is T_t×T_t training signal matrix and $\boldsymbol{N}$ is the T_r×T_t additive white Gaussian noise (AWGN) channel matrix. The MIMO channel matrix is considered to take the mathematical representation as $[\boldsymbol{C}_H]_{n,m}=r\varepsilon^{|n-m|}$; where r is a parameter which is fixed, ϵ is a parameter relating to correlation, n and m are indices of array of sensors which can be the antennas for transmitter and receiver [8].

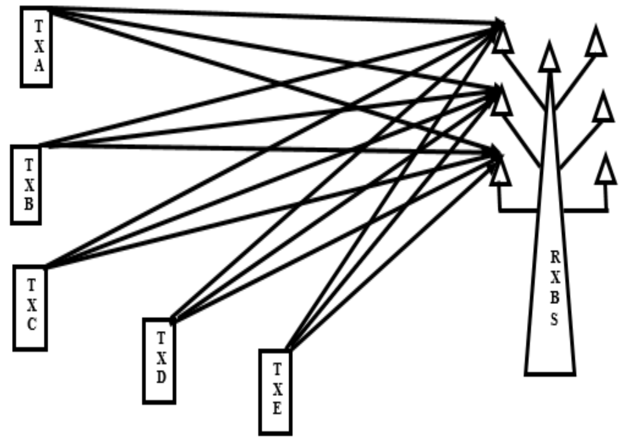

Figure 1. MIMO wireless transceiver system in the form of transmitter terminals and receiver base station

3. SIGNAL PROCESSING ALGORITHIMS FOR MEAN SQUARE ERROR FORMULATION

Mean square error can be formulated using signal processing algorithms such as least squares by considering the MIMO received signal matrix as stated earlier. To determine the mean square error for least squares (LS) signal processing algorithm the objective function is:

$$K = (\boldsymbol{R} - \boldsymbol{CD})^2 \tag{2}$$

Executing partial differentiation on considering the channel parameter $\boldsymbol{C}$ it can be given as:

$$\frac{\partial K}{\partial \boldsymbol{C}} = 2(\boldsymbol{R} - \boldsymbol{CD})(-\boldsymbol{D}) \tag{3}$$

To obtain its corresponding estimate of the estimation parameter $\boldsymbol{C}$ it is proceeded as:

$$0 = 2\left(\boldsymbol{R} - \widehat{\boldsymbol{C}}\boldsymbol{D}\right)(-\boldsymbol{D}) \tag{4}$$

On simplifying further the estimated value is:

$$\widehat{\boldsymbol{C}_{LS}} = \boldsymbol{RD}^{-1} \tag{5}$$

For the above given estimate the mean square error (MSE) pertaining to least squares signal processing algorithm are determined as per the representation:

$$MSE_{LS} = \left\{tr\left\{E\{(\boldsymbol{EE}^H)\}\right\}\right\} \tag{6}$$

where, $\boldsymbol{E}$ is the error matrix due to the least squares estimator, which is:

$$\boldsymbol{E} = \boldsymbol{C} - \widehat{\boldsymbol{C}}_{LS} \tag{7}$$

Substituting the value of error matrix in above MSE equation it is:

$$MSE_{LS} = \left\{ tr \left\{ E \left\{ \left((\boldsymbol{C} - \widehat{\boldsymbol{C}}_{LS})(\boldsymbol{C} - \widehat{\boldsymbol{C}}_{LS})^{H} \right) \right\} \right\} \right\} \tag{8}$$

Using the estimated value of LS signal processing algorithm in the above equation it is postulated as:

$$MSE_{LS} = \left\{ tr\left\{ E\{((\boldsymbol{C} - \boldsymbol{RD}^{-1})(\boldsymbol{C} - \boldsymbol{RD}^{-1})^{H})\} \right\} \right\} \tag{9}$$

In order to derive the MSE for minimum mean square error (MMSE) signal processing algorithm [8] it is $\boldsymbol{C}_{MMSE}$=$\boldsymbol{RV}_0$. where $\boldsymbol{V}_0 = argmin\ E\left\{ \left\| \boldsymbol{C} - \widehat{\boldsymbol{C}} \right\|^2 \right\}$ and it can also be written as $\boldsymbol{V}_0 = argmin\ E\{\|\boldsymbol{C} - \boldsymbol{RV}\|_F^2\}$ with respect to a matrix **V**. The estimation error is given as:

$$\varepsilon = E\{\|\boldsymbol{C} - \boldsymbol{RV}\|_F^2\} \tag{10}$$

Expanding further it is given as:

$$\begin{aligned} \varepsilon = tr\{\boldsymbol{R}_C\} - tr\{\boldsymbol{R}_C \boldsymbol{DV}\} - tr\{\boldsymbol{C}^H \boldsymbol{D}^H \boldsymbol{R}_C\} \\ + tr\{\boldsymbol{V}^{\boldsymbol{H}}(\boldsymbol{D}^H \boldsymbol{C}^H \boldsymbol{D} + \sigma_n^2 T_r \boldsymbol{I})\boldsymbol{V}\} \\ + tr\{V^H(D^H C^H D + \sigma_n^2 T_r I)V\} \end{aligned} \tag{11}$$

where,

$$\boldsymbol{V}_0 = (\boldsymbol{D}^H \boldsymbol{R}_C\ \boldsymbol{D} + \sigma_n^2 \boldsymbol{T}_r \boldsymbol{I})^{-1} \boldsymbol{D}^H \boldsymbol{R}_C \tag{12}$$

The MMSE signal processing algorithm is given as:

$$\boldsymbol{C}^{\wedge}_{MMSE} = \boldsymbol{R}((\boldsymbol{D}^H \boldsymbol{R}_C \boldsymbol{D} + \sigma_n^2 T_r \boldsymbol{I})^{-1} \boldsymbol{D}^H \boldsymbol{R}_C) \tag{13}$$

Based on the above representation estimate the mean square error (MSE) pertaining to MMSE signal processing algorithm is determined as per the representation:

$$MSE_{MMSE} = \left\{ tr\left\{ E\{(\boldsymbol{EE}^H)\} \right\} \right\} \tag{14}$$

where, E is the error matrix due to the MMSE signal processing algorithm, which is given as:

$$\boldsymbol{E} = \boldsymbol{C} - \widehat{\boldsymbol{C}}_{MMSE} \tag{15}$$

Substituting the value of error matrix in above MSE equation it reaches to:

$$MSE_{MMSE} = \left\{ tr \left\{ E \left\{ \left((\boldsymbol{C} - \widehat{\boldsymbol{C}}_{MMSE})(\boldsymbol{C} - \widehat{\boldsymbol{C}}_{MMSE})^{H} \right) \right\} \right\} \right\} \tag{16}$$

Now to derive maximum likelihood (ML) signal processing algorithm [9] for MIMO wireless transceiver for analysis of mean square error from equation (1) the probability density function (PDF) is defined as:

$$p(\boldsymbol{R}; \boldsymbol{C}) = \frac{1}{(2\pi)^{\frac{N}{2}} \det(\boldsymbol{C})}\ exp\left\{ -\frac{1}{2} (\boldsymbol{R} - \boldsymbol{CD})^T \boldsymbol{R}_c^{-1} (\boldsymbol{R} - \boldsymbol{CD}) \right\} \tag{17}$$

Proceeding to find the maximum likelihood estimate consider the objective function:

$$K(\boldsymbol{C}) = \left\{ (\boldsymbol{R} - \boldsymbol{CD})^T \boldsymbol{R}_c^{-1} (\boldsymbol{R} - \boldsymbol{CD}) \right\} \tag{18}$$

The above represents a quadratic function of elements of C and R_c^{-1} is a positive definite matrix on C. Taking partial differentiation with respect to C it is given as:

$$\frac{\partial ln}{\partial \boldsymbol{C}} = \frac{\partial}{\partial \boldsymbol{C}} \left\{ (\boldsymbol{R} - \boldsymbol{CD})^T \boldsymbol{R}_c^{-1} (\boldsymbol{R} - \boldsymbol{CD}) \right\} \tag{19}$$

On setting the gradient equal to zero, it reaches to:

$$\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \left(\boldsymbol{R} - \widehat{\boldsymbol{C}}_{ML} \boldsymbol{D} \right) = 0 \tag{20}$$

Multiplying further the above equation is:

$$\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{R} - \boldsymbol{D}^T \boldsymbol{R}_c^{-1} \widehat{\boldsymbol{C}}_{ML} \boldsymbol{D} = 0 \tag{21}$$

Rearranging to find the maximum likelihood estimation signal processing algorithm it is further given as:

$$\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{R} = \boldsymbol{D}^T \boldsymbol{R}_c^{-1} \widehat{\boldsymbol{C}}_{ML} \boldsymbol{D} \tag{22}$$

There ML estimation signal processing algorithm is:

$$\frac{\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{R}}{\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{D}} = \hat{C}_{ML} \tag{23}$$

Taking the denominator term to the numerator it is:

$$\left(\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{D} \right)^{-1}\ \boldsymbol{D}^T R_c^{-1}\ \boldsymbol{R} = \widehat{\boldsymbol{C}}_{ML} \tag{24}$$

Finally, the maximum likelihood estimation algorithm is given as:

$$\widehat{\boldsymbol{C}}_{ML} = \left(\boldsymbol{D}^T \boldsymbol{R}_c^{-1} \boldsymbol{D} \right)^{-1}\ \boldsymbol{D}^T \boldsymbol{R}_c^{-1}\ \boldsymbol{R} \tag{25}$$

Similar to representation of the estimate of LS and MMSE signal processing algorithm the mean square error (MSE) pertaining to ML signal processing algorithm is determined as per the representation:

$$MSE_{ML} = \left\{ tr\left\{ E\{(\boldsymbol{EE}^H)\} \right\} \right\} \tag{26}$$

where, $\boldsymbol{E}$ is the error matrix due to the ML signal processing algorithm, which is given as:

$$\boldsymbol{E} = \boldsymbol{C} - \widehat{\boldsymbol{C}}_{ML} \tag{27}$$

Substituting the value of error matrix in above MSE equation it is found to be as:

$$MSE_{ML} = \left\{ tr \left\{ E \left\{ \left((\boldsymbol{C} - \widehat{\boldsymbol{C}}_{ML})(\boldsymbol{C} - \widehat{\boldsymbol{C}}_{ML})^{H} \right) \right\} \right\} \right\} \tag{28}$$

4. SIMULATION RESULTS AND DISCUSSIONS

Simulation results for signal processing algorithms least squares(LS), minimum mean square error(MMSE) and maximum likelihood(ML) for mean square error (MSE) are obtained for MIMO wireless transceiver in matrix laboratory(MATLAB) against signal to noise ratio(SNR) at baseband level with simulation parameters shown in Table 1. Different transmitter antenna terminals to receiver base station with multiple antennas experiencing Rayeigh fading with elements zero mean complex Gaussian random variable and unit variance. The training data signals are elements from a discrete Fourier transform(DFT) matrix which are the simulation parameters and its impact provide significant reduction in MSE along with signal processing algorithms.

Table 1. Simulation parameters and metrics

Simulation Parameters	Metrics
Training signal	Elements of discrete Fourier transform matrix
MIMO wireless transceiver	$T_t=2;\ T_r=2;\ T_t=4;\ T_r=4;\ T_t=8;\ T_r=8$
Fading channel	MIMO Rayleigh fading channel
Signal processing Algorithms	Least squares, Minimum mean square error, Maximum likelihood
Evaluation entity	Mean square error

Figure 2 shows the mean square error (MSE) against signal to noise ratio (SNR) for least squares for different transmit antenna and receive antenna configurations of *2, 4* and *8*. To achieve a MSE of 10^{-3}, for *2,4* and *8* antennas it takes *24 dB, 18 dB* and *12 dB* for LS algorithm. The LS algorithm reduces the error in the data signal which are training sequence length from the DFT signal matrix which has entries of all signal unit vector length and complex signal conjugate entries. Further the correlation signal matrix provides correlation between different signal entities for different antenna configurations which contributes in obtaining reduced MSE performance.

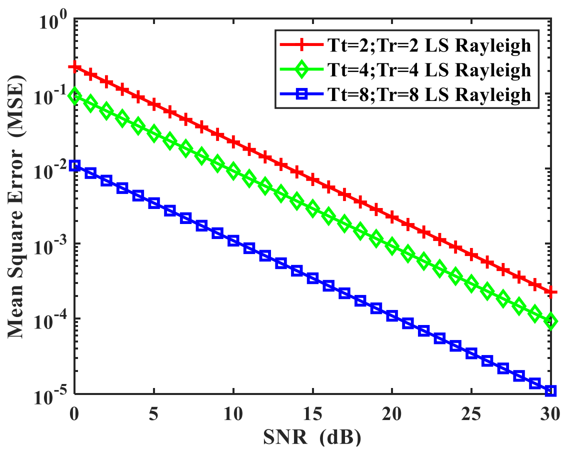

Figure 2. Mean square error vs SNR (dB) for least squares (LS) MIMO wireless transceiver in Rayleigh channel

Figure 3 shows the mean square error against signal to noise ratio (SNR) for MMSE different transmit antenna and receive antenna configurations of *2*, *4* and *8*. To achieve MSE of 10^{-3} for *2, 4* and *8* antennas it takes *27 dB, 17 dB* and *12 dB* for MMSE algorithm. The MMSE algorithm uses the statistics of the correlation matrix and conditional probability density function (PDF) to reach at the estimate and produces the mean square error. The MMSE signal processing algorithm has its computational complexity in terms of optimizing the conditional PDF to reduce error in lower SNR and higher SNR ranges.

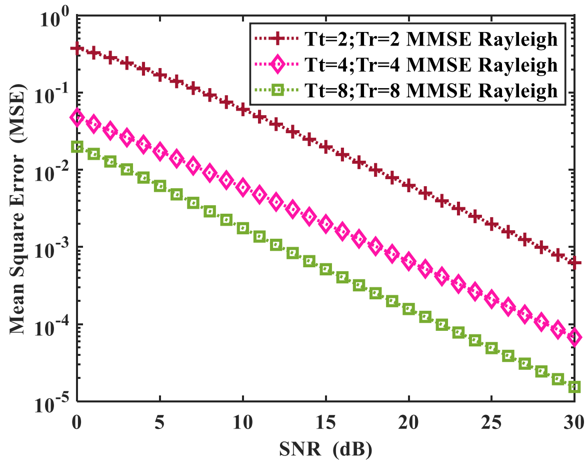

Figure 3. Mean square error vs SNR (dB) for minimum mean square error (MMSE) MIMO wireless transceiver in Rayleigh channel

Figure 4 shows the mean square error against signal to noise ratio (SNR) for ML different transmit antenna and receive antenna configurations of *2*, *4* and *8*. To achieve MSE of 10^{-3} for *2, 4* and *8* antennas it takes 1*8 dB, 16 dB* and *11 dB* for ML algorithm. The ML algorithm is a search algorithm which provides optimized value using the probability density function (PDF) to reach at the estimate of the MIMO wireless channel matrix and gives the mean square error value of the signal processing. The simulation results are given in Table 2.

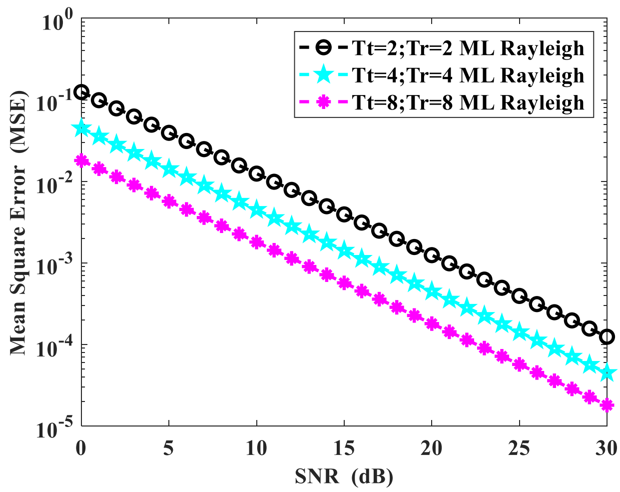

Figure 4. Mean square error vs SNR (dB) for maximum likelihood (ML) MIMO wireless transceiver in Rayleigh channel

Table 2. Signal processing algorithms with mean square error in MIMO wireless transceiver

MSE/Signal Processing Algorithms	Mean Square Error (MSE) 10^{-3}		
	$T_t=2;\ T_r=2;$	$T_t=4;\ T_r=4;$	$T_t=8;\ T_r=8$
LS	*24 dB*	*18dB*	*12dB*
MMSE	*27 dB*	*17dB*	*12dB*
ML	*18 dB*	*16 dB*	*11 dB*

Similar to the results obtained for Rayleigh MIMO fading channel mean square error, Rician fading MIMO channel has also been simulated for least squares signal processing algorithm and shown in Figure 5. The Rician fading channel has a line-of-sight (LoS) component which is considered as $K=5dB$ for obtaining the simulation results.

Figure 6 and Figure 7 show the mean square error performance for MIMO wireless transceiver using MMSE signal processing algorithm and ML signal processing algorithm obtained used Rician fading channel for the LoS component of $K=5$ dB. ML algorithm and MMSE algorithm exhibit better performance due to the statistical properties in relation to probability density function (PDF). The PDF pertains to the mean and variance of the random channel statistics. Further as an extension LS, MMSE and ML signal processing algorithms for MSE performance with error variances MIMO wireless transceivers can also be obtained.

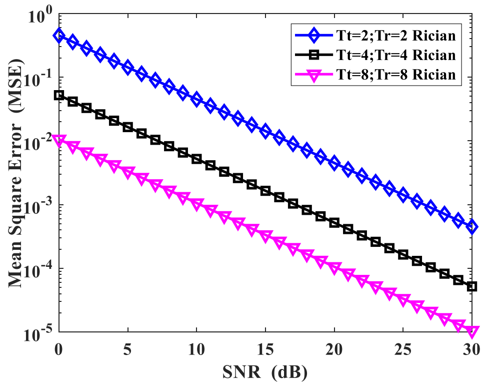

Figure 5. Mean square error vs SNR (dB) for least squares (LS) MIMO wireless transceiver in Rician channel

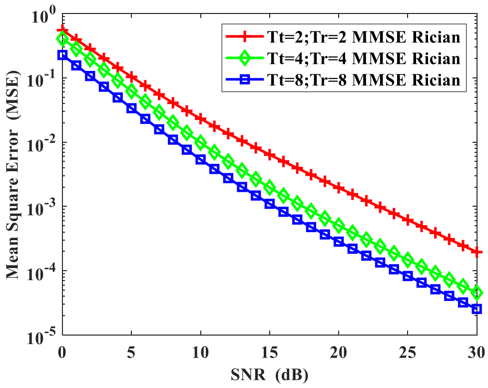

Figure 6. Mean square error vs SNR (dB) for minimum mean square error (MMSE) MIMO wireless transceiver in Rician channel

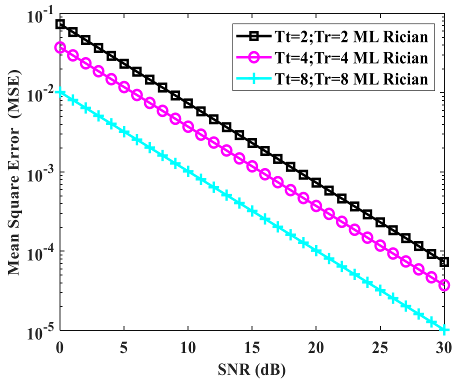

Figure 7. Mean square error vs SNR (dB) for maximum likelihood (ML) MIMO wireless transceiver in Rician channel

5. CONCLUSIONS

This research article concludes with the fact mean square error values for signal processing algorithms least squares (LS), minimum mean square error (MMSE) and maximum likelihood (ML) algorithms are obtained for MIMO wireless transceiver. Different antenna configurations in MIMO wireless transceiver are obtained and maximum likelihood (ML) algorithm provides the better performance in comparison to LS and MMSE signal processing algorithms with implications of theoretical results which are mathematically portrayed. From the simulation results extension work with future scope can be done to massive MIMO systems which are front runner technologies for the ongoing 5G and upcoming 6G wireless systems along with security aspects.

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