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Dusty Hydromagnetic Oldryod Fluid Flow in a Horizontal Channel with Volume Fraction and Energy Dissipation

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ABSTRACT

An unsteady magneto-hydrodynamics flow of dusty Oldroyd fluid through a horizontal channel has been investigated under the influence of dissipation of energy and volume fraction. This energy dissipation generates the mechanism of heat transfer in the governing fluid motion and the dust particles absorb the heat through conduction. To study the dusty-visco-elastic fluid flow, Saffman model and Oldroyd model have been used. The visco-elastic responses are exhibited through the two rheological parameters λ_1 & λ_2 (relaxation time and retardation time). To restrain the weak turbulent motion, a magnetic field of strength B_0 is applied along the transverse direction to the plate. The lower plate of the horizontal channel is kept fixed but the upper one is oscillating with the velocity $U_0(1+\varepsilon e^{i\omega' t'})$. The governing equations of motion are solved analytically and the results are discussed graphically/ numerically for various values of flow parameters involved in the solution.

Keywords: Oldroyd fluid, Saffman model, Nusselt number, Volume fraction, Relaxation and Retardation.

1. INTRODUCTION

Oldroyd [1, 2] has formulated the constitutive model to study the flow behaviour of visco-elastic fluids having two rheological parameters as relaxation time and retardation times characterizing the response of visco-elastic material. Sellers and Walker [3] have investigated the problem of liquid metal in an electrically insulated rectangular duct with a non-uniform magnetic field. Visco-elastic fluid flow past porous surface due to fluctuation in main flow has been analyzed by Mukhopadhyay and Chaudhury [4]. Rajagopal and Bhatnagar [5], Ray et al. [6] gave the exact solution of Oldroyd fluid flows. Exact solutions of (i) stokes problem, (ii) modified stokes problem, (iii) the time-periodic Poiseuille fow due to an oscillating pressure gradient (iv) the non-periodic fows between two boundaries, and (v) symmetric flow with an arbitrary initial velocity using Oldroyd model have been obtained by Hayat et al. [7]. Motion of electrically conducting, Oldroyd-B fluid between two non-conducting parallel plates in a rotating system under uniform transverse magnetic field has been studied by Hayat et al. [8]. Hall effects on unsteady fluid flows governed by Oldroyd-B model have been analyzed by Asghar et al. [9] and Hayat et al. [10]. Effect of Oldroyd fluid on unsteady free convective flow through porous medium along a moving porous hot vertical plate in presence of heat and mass transfer has been studied by Prasad et al. [11]. Unsteady hydro-magnetic flow of an Oldroyd fluid through a porous channel with oscillating walls using Laplace transform method has been examined by Ghosh [12]. Choudhury and Das [13], Choudhury Purkayastha [14], Choudhury and Das [15] have studied hydromagnetic visco-elastic fluid flow with various physical properties.

The combination of viscous fluid and dust particles is a subject of interest because of its occurrence in powder technology, transport of liquid slurries in chemical processing, nuclear processing and in different geophysical situations. Stability of laminar flow of dusty gas by neglecting the volume fraction of dust particles has been studied by Saffman [16]. Michael and Miller [17] have investigated the flow pattern of dusty gas. Rudinger [18] has generalized the problem of gas particle mixtures by considering volume fraction. Nayfeh [19] has formulated the equations of motion of dusty fluid mixtures in presence of volume fraction of dust particles. Gupta and Gupta [20] have examined the motion of a dusty gas with time varying pressure gradient. Analysis of flow pattern of unsteady dusty fluid through a rectangular channel with time dependent pressure gradient has been done by Singh [21]. An unsteady two dimensional flow of an electrically conducting dusty viscous fluid through a channel under the influence of transverse magnetic field has been studied by Singh and Ram [22]. Prasad and Ramacharyulu [23] have discussed the nature of a dusty incompressible fluid between two parallel surfaces under impulsive pressure gradient. Gupta and Gupta [24] have investigated the unsteady flow of a dusty non-Newtonian fluid through channel with volume fraction. Ajadi [25] has analyzed the isothermal flow of a dusty viscous electrically conducting fluid between oscillatory and non-oscillatory boundary motions. Unsteady Couette flow with heat transfer of a viscous incompressible electrically conducting fluid under the influence of an exponentially decreasing pressure gradient has been discussed by Attia et al. [26]. Kumar and Gupta [27] have discussed MHD forced convection and entropy generation of fluid flow though a circular channel having hyper porous medium. Kumar et al. [28] have studied flow problem between two horizontal parallel plates moving in opposite direction with radiation and mass transfer effects.

Application of visco-elastic fluids may be seen in various chemical and nucleaus industries, material processing, geophysics and in medical science. Flow problem of dusty visco-elastic fluid with heat transfer in presence of magnetic field may be applied in the extrusion of polymer sheet from a die[29]. Polymers are actually mixture of various organic solutions, so they may be modeled as visco-elastic fluid model (for fluid phase) and Saffman model (for dust phase). Another important application of dusty visco-elastic fluid flow model is in blood flow. In blood flow, some parts of energy transferred by heart are stored due to elasticity, some parts are transformed into heat by viscosity and remaining energy is used in motion of blood [30]. Viscosity in combination with elasticity plays an important role in blood flow. In this paper, an unsteady electrically conducting dusty visco-elastic flow characterized by Oldroyd fluid model through horizontal channel has been considered in presence of volume fraction and energy dissipation.

2. MATHEMATICAL FORMULATION

An unsteady flow of dusty electrically conducting Oldroyd fluid in a horizontal channel has been considered. The channel is bounded by two parallel plates, the lower one is kept fixed and the upper one is oscillating with velocity $u' = U_0(1+\varepsilon e^{i\omega' t'})$, U_0 is a constant and they are kept at different temperatures T_1 and $T_2 + \varepsilon(T_2 - T_1)e^{i\omega' t'}$, ($T_2 > T_1$). The upper plate is oscillating about the mean temperature T_2. Dust particles are assumed to be electrically non-conducting, spherical in shape and uniformly distributed throughout the fluid. A magnetic field of uniform strength B_0 is applied along the transverse direction to the plate.

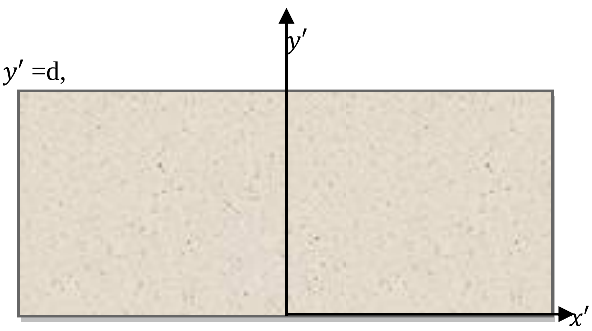

Figure 1. Physical description of the Problem

Equation of Continuity:

$$\mathrm{v}_{i,i} = 0 \tag{1}$$

Equation of Continuity for dust particles:

$$\mathrm{v}_{pi,i} = 0 \tag{2}$$

Momentum Equation:

$$\rho\left[\frac{\partial \mathrm{v}_i}{\partial t} + 2\mathrm{v}_k\frac{\partial \mathrm{v}_i}{\partial x_k}\right] = -p_{,i} + \tau_{ij,j} + \varepsilon_{ijk}J_jB_k + \frac{KN}{1-\phi}\left(\mathrm{v}_{pi} - \mathrm{v}_i\right) \tag{3}$$

Momentum equation for dust particles:

$$m_p\left[\frac{\partial \mathrm{v}_{pi}}{\partial t} + \mathrm{v}_{pk}\frac{\partial \mathrm{v}_{pi}}{\partial x_k}\right] = \frac{\phi}{\rho}\left(-p_{,i} + \tau_{ij,j} + \varepsilon_{ijk}J_jB_k\right) - K\left(\mathrm{v}_{pi} - \mathrm{v}_i\right) \tag{4}$$

Constitutive Equation:

$$\left(1 + \lambda_1\frac{d}{dt}\right)\tau_{ik} = 2\eta_0\left(1 + \lambda_2\frac{d}{dt}\right)\epsilon^{ik} \tag{5}$$

where, λ_1 and λ_2 denote relaxation and retardation times respectively. ($\lambda_1 = 0, \lambda_2 = 0$) characterizes Newtonian fluid, ($\lambda_1 = 0, \lambda_2 \neq 0$) characterizes Second-grade fluid and ($\lambda_1 \neq 0, \lambda_2 = 0$) represents the Maxwell fluid model.

Energy Equation:

$$\rho C_p\frac{\partial T'}{\partial t'} = kT'_{,ii} + \tau_{ij}v_{i,j} + \frac{\rho_p C_s}{\gamma_T}\left(T'_p - T'\right) \tag{6}$$

Following Attia et al. [26], energy Equation for dust particles:

$$\frac{\partial T'_p}{\partial t'} = \frac{1}{\gamma_T}\left(T' - T'_p\right) \tag{7}$$

2.1 Velocity distribution

For an incompressible unsteady flow through a horizontal channel, the stress components derived from (5) are obtained as follows:

$$\tau'_{xx} + \lambda_1\left(\frac{\partial}{\partial t'}\tau'_{xx} - \frac{\partial u'}{\partial y'}2\tau'_{yx}\right) = \frac{-\eta_0\lambda_2}{2}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{8}$$

$$\tau'_{yx} + \lambda_1\left(\frac{\partial}{\partial t'}\tau'_{xy} - \frac{\partial u'}{\partial y'}\tau_{yy}\right) = \eta_0\left[1 + \lambda_2\left(\frac{\partial}{\partial t'}\right)\right]\frac{\partial u'}{\partial y'} \tag{9}$$

$$\tau_{yy} + \lambda_1\left(\frac{\partial}{\partial t}\tau_{yy}\right) = 0 \tag{10}$$

Solving (10), we get $\tau_{yy} = 0$ [8, 10] and the governing equations of motion are:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \frac{1}{\rho}\frac{\partial \tau'_{xy}}{\partial y'} - \frac{\sigma B_0^2 u'}{\rho} + \frac{KN}{1-\phi}(v' - u') \tag{11}$$

$$\frac{\partial v'}{\partial t'} = -\frac{\phi}{\rho m_p}\left[\frac{\partial p'}{\partial x'} - \frac{\partial \tau'_{xy}}{\partial y'} + \sigma B_0^2 u'\right] - KN(v' - u') \tag{12}$$

The boundary conditions of the problem are

$$y = 0: u' = 0, v' = 0;$$

$$y = d:\ u' = U(t) = U_0\left(1 + \varepsilon e^{i\omega' t'}\right) = v' \tag{13}$$

Using the constitutive equations, the equations (11) and (12) are written as follows:

$$\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial x'} + \frac{1}{\rho}\eta_0\left[\frac{\partial u'}{\partial y'} + \lambda_2\left(\frac{\partial^2 u'}{\partial y'\partial t}\right)\right] - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)u' + \frac{KN}{1-\phi}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)(v'-u') \quad (14)$$

$$\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial v'}{\partial t'} = \frac{\phi}{m_p}\left[-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial x'} + \frac{1}{\rho}\eta_0\left\{\frac{\partial u'}{\partial y'} + \lambda_2\left(\frac{\partial^2 u'}{\partial y'\partial t}\right)\right\} - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)u'\right] + \frac{KN}{m_p}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)(u'-v') \quad (15)$$

The pressure gradient terms are eliminated by using boundary conditions (13) and it is observed that

$$-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial x'} = (A_1+iA_2)e^{i\omega' t'} \quad (16)$$

where, A_1 and A_2 are arbitrary constants.
Using (16) in (14) and (15), we get

$$\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial u'}{\partial t'} = (A_1+iA_2)e^{i\omega' t'} + \frac{1}{\rho}\eta_0\left[\frac{\partial u'}{\partial y'} + \lambda_2\left(\frac{\partial^2 u'}{\partial y'\partial t}\right)\right] - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)u' + \frac{KN}{1-\phi}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)(v'-u') \quad (17)$$

$$\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial v'}{\partial t'} = \frac{\phi}{m_p}\left[(A_1+iA_2)e^{i\omega' t'} + \frac{1}{\rho}\eta_0\left\{\frac{\partial u'}{\partial y'} + \lambda_2\left(\frac{\partial^2 u'}{\partial y'\partial t}\right)\right\} - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)u'\right] + \frac{KN}{m_p}\left(1+\lambda_1\frac{\partial}{\partial t'}\right)(u'-v') \quad (18)$$

Let us introduce the following non-dimensional quantities

$$u=\frac{u'}{U_0}, v=\frac{v'}{U_0}, y=\frac{y'}{d}, \frac{t}{t'}=\frac{U_0}{d}, \frac{\omega}{\omega'}=\frac{d}{U_0}, R=\frac{U_0 d}{\nu}, \alpha_1=\frac{\lambda_1 U_0}{d}$$
$$\alpha_2=\frac{\lambda_2 U_0}{d}, M=\frac{\sigma B_0^2 d}{\rho U_0}, \epsilon_1=\frac{1}{1-\phi}, f=\frac{KNd^2}{\eta_0}, G=\frac{m_p\eta_0}{KNd^2} \quad (19)$$

Using the above mentioned non-dimensional quantities in (17) and (18), we get the following dimensionless equations of motion,

$$\frac{\partial u}{\partial t} + \alpha_1\frac{\partial^2 u}{\partial t^2} = (A_1+iA_2)e^{i\omega t} + \frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2} + \alpha_2\frac{\partial^3 u}{\partial y^2\partial t}\right) - \frac{1}{R}\left(1+\alpha_1\frac{\partial}{\partial t}\right)[Mu - \epsilon_1 f(v-u)] \quad (20)$$

$$\frac{\partial v}{\partial t} + \alpha_1\frac{\partial^2 v}{\partial t^2} = \frac{\phi}{m_p}\left[(A_1+iA_2)e^{i\omega t} + \frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2} + \alpha_2\frac{\partial^3 u}{\partial y^2\partial t}\right) - \frac{1}{R}\left(1+\alpha_1\frac{\partial}{\partial t}\right)\left[u - \frac{u-v}{G}\right]\right] \quad (21)$$

2.2 Temperature distribution

In the governing fluid motion, the energy equations of fluid and dust particles in Cartesian form are given below:

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + \tau'\frac{\partial u}{\partial y} + \frac{\rho_p C_s}{\gamma_T}(T'_p - T') \quad (22)$$

$$\frac{\partial T'_p}{\partial t'} + \frac{1}{\gamma_T}(T'_p - T') = 0 \quad (23)$$

The relevant boundary conditions are

$$y=0: T'=T'_p=T_1;$$
$$y=d:\ T'=T'_p=T_2+\varepsilon(T_2-T_1)e^{i\omega' t'} \quad (24)$$

Introducing the dimensionless variables

$$T=\frac{T'-T_1}{T_2-T_1}, T_p=\frac{T'_p-T_1}{T_2-T_1}, Pr=\frac{\eta_o C_p}{k},$$
$$Ec=\frac{U_0^2}{C_p(T_2-T_1)}, l_1=\frac{\rho_p}{\rho}, l_2=\frac{C_s}{C_p}, L_0=\frac{d}{U_0\gamma_T}$$

into (22) and (23) we get,

$$\frac{\partial T}{\partial t} = \frac{1}{RPr}\frac{\partial^2 u}{\partial y^2} + Ec\tau\frac{\partial u}{\partial y} + l_1 l_2 L_0(T_p - T) \quad (25)$$

$$\frac{\partial T_p}{\partial t} + L_0(T_p - T) = 0 \quad (26)$$

where Pr is the Prandtl number, Ec be the Eckert number, L_0 is the temperature relaxation time parameter in dimensionless form.

The dimensionless boundary conditions for solving the equations (20), (21) and (25), (26) are

$$\left.\begin{array}{r} y=0: u=0=v=T=T_p \\ y=d: u=1+\varepsilon e^{i\omega t}=v, T=T_p=1+\varepsilon e^{i\omega t} \end{array}\right\} \quad (27)$$

3. METHOD OF SOLUTION

Assuming small amplitude of oscillation, we represent the velocity and temperature of fluid and dust particles as

$$u=u_1+\varepsilon e^{iwt}u_2+o(\varepsilon^2),\ v=v_1+\varepsilon e^{iwt}v_2+o(\varepsilon^2) \quad (28)$$

$$\left.\begin{array}{r} T=T_1+\varepsilon e^{iwt}T_2+o(\varepsilon^2) \\ T_p=T_{p1}+\varepsilon e^{iwt}T_{p2}+o(\varepsilon^2) \end{array}\right\} \quad (29)$$

Using (28) in (20) and (21) and equating the like terms, we get

$$-u_0'' + (M+\varepsilon_1 f)u_0 = \varepsilon_1 f v_0 \quad (30)$$

$$iu_1\omega - \alpha_1 u_1 \omega^2 = \frac{(A_1 + iA_2)}{\varepsilon} + \frac{u_1''}{R} + i\alpha_2 \frac{\omega u_1''}{R} - (1 + i\alpha_1\omega)\left[\frac{Mu_1}{R} + \frac{\varepsilon_1 f}{R}(v_1 - u_1)\right] \quad (31)$$

$$\varepsilon_2 G(u_0'' + Mu_0) = v_0 \quad (32)$$

$$v_1\omega(i - \alpha_1\omega) = \varepsilon_2\left[\frac{(A_1 + iA_2)}{\varepsilon} + \frac{u_1''(1 + i\alpha_2\omega)}{R} - \frac{M(1 + i\alpha_1\omega)u_1}{R}\right] - \frac{(1 + i\alpha_1\omega)(v_1 - u_1)}{GR} \quad (33)$$

where, $\varepsilon_2 = \frac{\phi}{m_p}$.

Viscous drags are formed at the surfaces of the channel and its non-dimensional form is given by

$$\alpha_1 \frac{\partial \tau}{\partial t} + \tau = \frac{1}{R}\frac{\partial u}{\partial y} + \frac{\alpha_2}{R}\frac{\partial^2 u}{\partial y \partial t}$$

Solving the above linear differential equation, we get

$$\tau = \frac{\sqrt{A_{33}}}{R}\left(C_3 e^{\sqrt{A_{33}}y} - C_4 e^{-\sqrt{A_{33}}y}\right)\left(1 - e^{-\frac{t}{\alpha_1}}\right) + \frac{\varepsilon\sqrt{A_{27}}}{\left(\frac{1}{\alpha_1} + i\omega\right)}\left(C_1 e^{\sqrt{A_{27}}y} - C_2 e^{-\sqrt{A_{27}}y}\right)\left(e^{i\omega t} - e^{-\frac{t}{\alpha_1}}\right) + \frac{i\alpha_2\omega\varepsilon\sqrt{A_{27}}}{\alpha_1 R\left(\frac{1}{\alpha_1} + i\omega\right)}\left(C_1 e^{\sqrt{A_{27}}y} - C_2 e^{-\sqrt{A_{27}}y}\right)\left(e^{i\omega t} - e^{-\frac{t}{\alpha_1}}\right) \quad (34)$$

Using (34) and (29) in (25) and (26), and equating the like powers of ε and neglecting the higher powers, we get the following ordinary differential equations:

$$\frac{T_1''}{RPr} + \frac{Ec\sqrt{A_{33}}}{R}\left(C_3^2 e^{\sqrt{A_{33}}y} - C_3^4 e^{-\sqrt{A_{33}}y}\right) + l_1 l_2 L_0\left(T_{p1} - T_1\right) = 0 \quad (35)$$

$$\frac{1}{RPr}T_2'' + Ec\left[\frac{\sqrt{A_{27}}}{\left(\frac{1}{\alpha_1} + i\omega\right)}\left(C_1 e^{\sqrt{A_{27}}y} - C_2 e^{-\sqrt{A_{27}}y}\right)\left(C_3 e^{\sqrt{A_{33}}y} + C_4 e^{-\sqrt{A_{33}}y}\right) + \frac{i\alpha_2\omega\varepsilon\sqrt{A_{27}}}{\alpha_1 R\left(\frac{1}{\alpha_1} + i\omega\right)}\left(C_1 e^{\sqrt{A_{27}}y} - C_2 e^{-\sqrt{A_{27}}y}\right)\left(C_3 e^{\sqrt{A_{33}}y} + C_4 e^{-\sqrt{A_{33}}y}\right) + \frac{\sqrt{A_{33}}}{R}\left(C_3 e^{\sqrt{A_{33}}y} - C_4 e^{-\sqrt{A_{33}}y}\right)\left(C_1 e^{\sqrt{A_{27}}y} + C_2 e^{-\sqrt{A_{27}}y} + A_{28} + iA_{29}\right)\right] + l_1 l_2 L_0\left(T_{p2} - T_2\right) = i\omega T_2 \quad (36)$$

$$T_{p1} = L_0\left(T_1 - T_{p1}\right) \quad (37)$$

$$T_{p2} = L_0\left(T_2 - T_{p2}\right) \quad (38)$$

Solving the above equations, subject to the boundary conditions (27), the temperature profile of fluid and dust particles are obtained from (29) and the rate of heat transfer is given by

$$Nu = -\left.\frac{\partial T}{\partial y}\right|_{y=0 \ or \ 1}$$

4. DISCUSSIONS

A problem of unsteady dusty electrically conducting Oldroyd fluid flow through the horizontal channel has been studied in presence of volume fraction and energy dissipation due to viscosity. This dissipation of energy creates heat transfer along with the conduction of heat from the surface to the fluid motion. The results are discussed for various pair of values of α_1, α_2 and volume fraction φ. Velocity profile, temperature fields, skin frictions and rate of heat transfer are analyzed numerically and graphically for various values flow parameters present in the solution. Figures 2 to 4 represent the pattern of velocity profiles of fluid and dust particles against the displacement variable and figure 5 shows the pattern of temperature field of governing fluid and dust particles. In figures 2 to 4, the horizontal axis corresponds to the displacement variable y and the vertical axis corresponds to the velocity. Similarly, in figure 5, vertical axis corresponds to the temperature and horizontal axis indicates the displacement variable.

It is seen that (figure 2), velocity of fluid particles rises with the increasing value of y and the maximum speed is noticed in the neighbourhood of the upper plate, which is oscillating about a non-zero mean velocity U_0 and dust particles experience a back flow in the neighbourhood of the lower fixed plate and then gradually its magnitude rises towards the upper plate but in-comparison to the fluid particles, the dust particles are lacking behind along the increasing values of displacement variable (y).

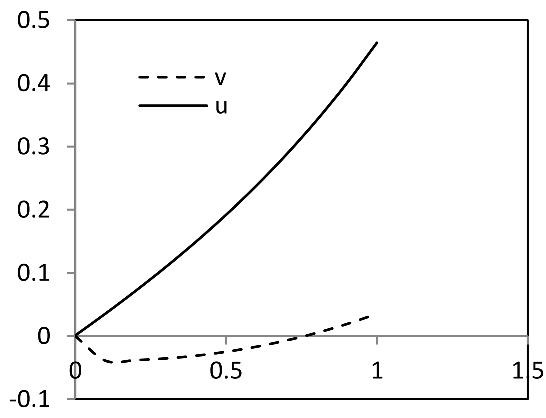

Figure 2. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Table 1 is showing the nature of fluid motion for different cases of α_1 and α_2. For smaller values of these two rheological parameters, the energies used in visco-elastic responses are of smaller order of magnitude and hence maximum energy can be reserved and as a response the fluid flows experience acceleration in the motion, i.e, increasing values of α_1 and α_2 decelerates the fluid motion. Effect of volume fraction on fluid motion and dust particles are shown in figure 3 and the figures enable the fact that the presence of volume fraction accelerates the fluid motion. Its effect is more prominent on the motion of dust particles, as the magnitude of the speed of dust particles increases but the dust particles experience a back flow in the neighborhood of the lower plate. Figure 4 states that the motion of dust particles reaches steady state quickly then in-comparison to the fluid particles as the change in time creates a negligible variation in the motion of dust particles.

Table 1. M=2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	u (α1=0.2, α2=0.2)	u (α1=0.2, α2=0.5)	u (α1=0.5, α2=0.2)
y=0	0.0010	0.001	0.001
y=0.1	0.0355	0.035	0.0348
y=0.2	0.0715	0.0704	0.0702
y=0.3	0.1093	0.1076	0.1074
y=0.4	0.1493	0.1472	0.1468
y=0.5	0.1920	0.1895	0.1891
y=0.6	0.2379	0.235	0.2345
y=0.7	0.2875	0.2842	0.2838
y=0.8	0.3414	0.3377	0.3373
y=0.9	0.4001	0.3962	0.3957
y=1	0.4644	0.4601	0.4596

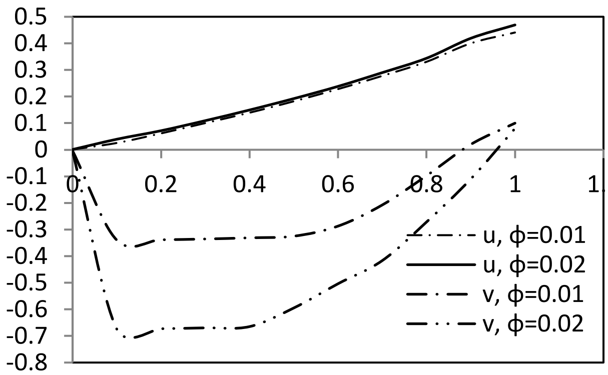

Figure 3. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

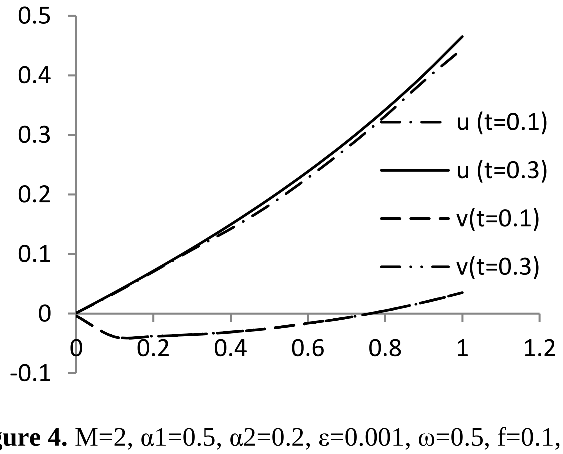

Figure 4. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, mp=0.2, φ=0.01, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

After studying the velocity profile, now the viscous drag at the surfaces formed by the fluid motion are calculates for various values of flow parameters and it is represented in tabular form. Table 2, states that, magnitude of shearing stresses at both the plates increase with the time but the effect of time is seen prominent at the upper plate.

Effect of relaxation time and retardation time on shearing stress are seen in Table 3a and Table 3b and it is noticed that increase of the relaxation time diminishes the magnitude the of shearing stress at the lower plate.

Table 2. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	τ (lower plate) (φ=0.01)	τ (upper plate) (φ=0.01)	τ (lower plate) (φ=0.02)	τ (upper plate) (φ=0.02)
t=0.1	2.9325	9.1069	2.9326	9.1074
t=0.2	3.8135	11.5578	3.8136	11.5585
t=0.3	4.6956	13.9910	4.6958	13.9918
t=0.4	5.5766	16.4003	5.5769	16.4012
t=0.5	6.4544	18.7796	6.4547	18.7807
t=0.6	7.3266	21.1232	7.3271	21.1244
t=0.7	8.1912	23.4250	8.1917	23.4264
t=0.8	9.0460	25.6794	9.0465	25.6809
t=0.9	9.8888	27.8807	9.8894	27.8824
t=1	10.7175	30.0234	10.7182	30.0252

Table 3a. M=2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2 l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	(α1=0.2, α2=0.2)	(α1=0.2, α2=0.5)	(α1=0.5, α2=0.2)
	τ (lower plate)	τ (lower plate)	τ (lower plate)
0.1	4.8329	5.6635	2.9325
0.2	5.7999	8.0125	3.8135
0.3	6.7631	10.3520	4.6956
0.4	7.7200	12.6763	5.5766
0.5	8.6683	14.9796	6.4544
0.6	9.6055	17.2561	7.3266
0.7	10.5294	19.5001	8.1912
0.8	11.4376	21.7060	9.0460
0.9	12.3279	23.8683	9.8888
t=1	13.1980	25.9816	10.7175

Table 3b. M=2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	(α1=0.2, α2=0.2)	(α1=0.2, α2=0.5)	(α1=0.5, α2=0.2)
	τ (upper plate)	τ (upper plate)	τ (upper plate)
0.1	7.5761	5.6635	9.1069
0.2	10.3098	8.0125	11.5578
0.3	13.0288	10.3520	13.9910
0.4	15.7263	12.6763	16.4003
0.5	18.3956	14.9796	18.7796
0.6	21.0300	17.2561	21.1232
0.7	23.6230	19.5001	23.4250
0.8	26.1680	21.7060	25.6794
0.9	28.6587	23.8683	27.8807
1	31.0888	25.9816	30.0234

The same diminishing effect of retardation time is seen on the shearing stress at the upper plate but it has an increasing effect at the lower plate.Volume fraction also has a positive impact on shearing stresses at both the plates but its effect is seen superior at the upper plate (Table 2).

Both the temperatures of fluid particles and dust particles rise in the neighbourhood of the upper oscillating plate (figure 5). Effects of relaxation time and retardation time on the temperature fields are represented by Table 4 and Table 5 and it is experienced that during the growth of relaxation time, the temperature of dusty visco-elastic fluid and dust particles experience enhancing pattern over the entire channel but a reverse mechanism is seen during the growth of retardation parameter.

Nusselt number plays an important role in the mechanism of heat transfer as it enables the rate of heat transfer in the governing fluid motion. Table 6 is representing the numerical values of Nusselt number at the plates for a time period [0.1, 1] and it can be concluded that magnitude of rate of heat transfer increases in the above mentioned time period at both the plates.

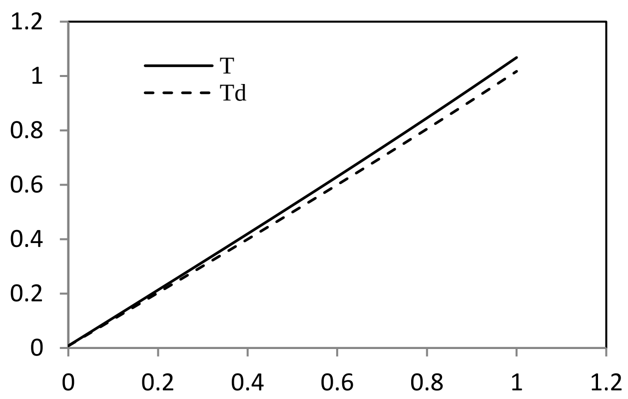

Figure 5. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Table 4. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	T (α1=0.2, α2=0.2)	T (α1=0.2, α2=0.5)	T (α1=0.5, α2=0.2)
y=0	0.0078	0.0077	0.008
y=0.1	0.1102	0.1101	0.1104
y=0.2	0.2128	0.2127	0.2131
y=0.3	0.3159	0.3158	0.3161
y=0.4	0.4196	0.4195	0.4199
y=0.5	0.5242	0.5241	0.5245
y=0.6	0.6299	0.6298	0.6302
y=0.7	0.7369	0.7368	0.7372
y=0.8	0.8454	0.8453	0.8457
y=0.9	0.9557	0.9556	0.956
y=1	1.0679	1.0678	1.0682

Table 5. M=2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	T_p(α1=0.2 ,α2=0.2)	T_p(α1=0.2,α2= 0.5)	T_p(α1=0.5,α2= 0.2)
y=0	0.0074	0.0073	0.0076
y=0.1	0.1049	0.1048	0.1052
y=0.2	0.2027	0.2026	0.2029
y=0.3	0.3008	0.3007	0.3011
y=0.4	0.3996	0.3995	0.3999
y=0.5	0.4992	0.4991	0.4995
y=0.6	0.5999	0.5998	0.6002
y=0.7	0.7018	0.7017	0.7021
y=0.8	0.8052	0.8051	0.8054
y=0.9	0.9102	0.9101	0.9105
y=1	1.0171	1.0170	1.0174

Table 6. M=2, α1=0.5, α2=0.2, ε=0.001, ω=0.5, f=0.1, G=0.8, R=0.2, φ=0.01, mp=0.2, l_1=0.5, l_2=0.5, Pr=7, Ec=0.01, Lo=0.05

Cases	Nu (lower plate)	Nu (upper plate)
t=0.1	0.9504	0.9349
t=0.2	0.9507	0.9353
t=0.3	0.9511	0.9369
t=0.4	0.9518	0.9367
t=0.5	0.9526	0.9378
t=0.6	0.9536	0.09391
t=0.7	0.9548	0.9407
t=0.8	0.9562	0.9424
t=0.9	0.9577	0.9444
t=1	0.9594	0.9467

7. CONCLUSIONS

Some of the important conclusions from the above work are cited below:

- Speed of fluid flow is maximum in the neighbourhood of the upper plate.
- Dust particles experience a back flow in the neighbourhood of the lower plate.
- Increasing values of relaxation and retardation time parameters decelerates the fluid motion.
- Temperature of fluid and dust particles increases uniformly from lower surface to upper surface.
- Magnitude of Nusselt number increases in the time interval [0.1, 1] at both the plates.

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NOMENCLATURE

v_i, u'	Velcoity of fluid, LT^{-1}
v_{pi}, v'	Velocity of dust in tensorial form, LT^{-1}
x_i, x'	Displacement variable, L
t'	Time, T
p	Fluid pressure, $ML^{-1}T^{-2}$
d	Distance between two plates, L
J_i	Current density, IL^{-2}
K= 6πμa	Stokes constant, MT^{-1}
T'	Temperature of fluid, K(Kelvin)
$T'p$	Temperature of dust particles, K
C_p	Specific heat of fluid at constant pressure, $L^2T^{-2}K^{-1}$
C_p	Specific heat of dust at constant pressure, $L^2T^{-2}K^{-1}$
N	Number of dust particles per unit volume, L^{-3}
m_p	Average mass of dust particles, M
k	Thermal conducitivity, $MLT^{-3}I^2$
B_i	Magnetic induction vector, $MT^{-2}I^{-1}$
U_0	A constant, LT^{-1}
y	Dimensionless displacement variable
u	Dimensionless velocity of fluid
v	Dimensionless velocity of dust particles
t	Dimensionless time
T	Dimensionless temperature of fluid
T_p	Dimensionless temperature of dust
f	Particle concentration parameter
R	Reynolds number
M	Hartmann number
G	Particle mass parameter
Pr	Prandtl number
Ec	Eckert number
Nu	Nusselt number
Sh	Shearing stress
L_0	Dimensionless temperature relaxation time

Greek symbols

ρ	Density of fluid, ML^{-3}
ρ_0	Density of dust particle, ML^{-3}
τ_{ij}, τ	Viscous stress, $ML^{-1}T^{-2}$
ν	Kinematic viscosity, L^2T^{-1}
η_0	Dynamic viscosity, $ML^{-1}T^{-1}$
σ	Electrical conductivity, $L^{-3}M^{-1}T^3I^2$
ε	Dimensionless amplitude of oscillation
λ_1	Relaxation time parameter, T
λ_2	Retardation time parameter, T
α_1	Dimensionless relaxation time
α_2	Dimensionless retardation time
ϕ	Volume fraction
γ_T	Temperature relaxation time, T
ω'	Frequency of oscillation, T^{-1}
ω	Dimensionless frequency
ε_{ijk}	Levi-Civita symbol